Optimal Power Allocation for Multichannel Cognitive Radio Systems Using Stackelberg Game

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Abstract – We consider a non-orthogonal multiple access scheme e.g. CDMA based cellular multi-channel cognitive radio network (CRN). Here multiple secondary users share a common set of channels simultaneously. For such an environment, we develop a distributed power allocation approach based on a game theoretic formulation among secondary users to satisfy their rate requirement. We model the problem using a two-stage leader-follower game known as Stackelberg game where base station and users are considered as leader and followers respectively. Since the spectrum is licensed to a primary network, base stations of the cellular secondary network should protect primary users against interference caused by secondary communications. We are interested for downlink power allocation with pricing that maximizes utilities of both BS and CRs in presence of primary network. A nonlinear relationship which is developed between bit rate and SINR is solved by game theory. Simulation results show that our proposed power control scheme provides higher SINR levels (or equivalently higher data rates) while consuming lower average transmit power of secondary users.

Keywords–Cognitive radio network(CRN), power allocation, non-cooperative game, Stackelberg game, Nash equilibrium(NE), pricing, QAM, ASK, PSK and BER.

I. INTRODUCTION

Power control of users of cognitive radio is a critical issue. Spectral efficiency and Energy efficiency are also interdependent with power control. Decreasing power consumptions in devices of cognitive radios affects quality of services e.g. bit error rate. So adaptive power control scheme is required. Earlier solutions are Calculus based approach, Convex Optimization, Karush-Khun-Tucker solution, Variational-inequality etc[1]. Each of the method have many limitations. Such as they are (a) time consuming (b) large solution space and (c) failure to solve nonlinear constraints. Game theory can substantially solve the above problems. It can also converge the solution in short time. Game has generally two forms non-cooperative and cooperative.

In this paper we apply Stackelberg non-cooperative game for power control of cellular cognitive radio network. Players of Stackelberg game classified as leaders and followers. Here a base station and secondary cognitive radios in each cell is considered as a leader and followers respectively. Leader(BS) move first and followers are secondary cognitive radios(CRs)[2]. Consequently each of BSs have more benefit than follower. This paradigm best suits for cells of cognitive radio networks. We are interested in developing a decentralized scheme for downlink power allocation with pricing that maximizes utilities of both BS and CRs in presence of primary network. Each BS wants to earn higher income from the CRs in its cell while spending less transmit power. Each CR is interested in gaining higher level of SINR (or equivalently data rate) with high QoS (e.g. bit error rate) while paying less to the associated BS. With the objective of maximizing utilities and upper bound of the power of secondary cognitive radios are such that upper bound of bit error rate(BER) is maintained and primary user can tolerate interference limit.

II. EXSISTING WORK

Nowadays application of Game theory in wireless communication is one of the substantial matter. Previous method of calculus is restricted in finding critical point by avoiding point of inflection. Convex Optimization, Karush-Khun-Tucker solution methods are applied to reduce search space and to find optimal solution avoiding suboptimal solutions[1]. The above methods fail if any nonlinearity exists between two parameters.

Power allocation of cognitive radio network has generally two forms: Centralized architecture and game theory based distributed. A centralized architecture which requires more control signals and parameters can difficult to control and computationally hard. A user based distributed architecture easy to control but sometimes gives suboptimal result[3]. However, the latter more preferable as it is computationally easier and requires less power thus also energy efficient.

Power of spectrum sensing and power allocation of nodes are considered now simultaneously. In [4], the authors formulated as a distributed non-cooperative power...
allocation game to maximize the total throughput of the cognitive radio users thus optimizing jointly both the detection operation and the power allocation. They took into account the influence of the sensing accuracy. In [5] the authors proposed an algorithm based on the VCG (Vickrey–Clarke–Groves) model in a non cooperative game for spectrum allocation. They did this between secondary transmissions that guarantee a required minimum data rate for both PUs and SUs, assuming a fixed value of the bit error rate. In [6], the authors considered a sensing-based spectrum sharing scenario in cognitive radio networks where the overall objective is to maximize the sum-rate of each cognitive radio user by optimizing jointly both the detection operation based on sensing and the power allocation. They took account the influence of the sensing accuracy and the interference limitation to the primary users. The resulting optimization problem for each cognitive user is non-convex, thus leading to a non-convex game. Thus they presented a new challenge when analyzing the equilibria of this game where each cognitive user represents a player.

At present Stackelberg non-cooperative game is widely used for resource allocation of cognitive radio network. Stackelberg game has three forms. They are as follows: (a) multiple leaders and multiple followers, (b) single leader and multiple followers and (c) single leader and single follower. Our problem is best suitable with second form[2]. Because in each cell of cognitive radio network there is one base station (BS) and multiple cognitive radios (CRs).

In [7] the authors addressed the application of game theory to resource management of a cognitive network in the downlink side. As the optimum resource allocation problem is NP-hard, they used game theory to propose a distributed scheme for power allocation to guarantee communications of primary networks and provide required signal-to-noise plus interference of cognitive radios. In return, each base station charges the cognitive radios in its cell for their provided received SINR. They modelled the problem using a two-stage Stackelberg game.

From engineering point of views cellular type network is feasible. It has less information overhead[8]. Additional information theoretic facilities is added to such a network. We consider multiuser with multichannel in each cell of cognitive radio network. Co-exist of multiple user in a channel increases spectral efficiency. Power control should meet the following requirements: (a) Secondary users rate requirements: Power consumption should such that users’ total rate be greater than a specified limit thus poorer channels get benefit. (b) Channels’ Rate requirement: Power consumption should such that channels’ total rate be less than a specified limit. (c) Channels’ power requirement: Power consumption should such that channels’ total power should be less than a specified limit such that primary users can tolerate interference[9]. All these requirements are often contradictory with each other.

We solve this problem by applying Stackelbergh game and gets equilibrium named as Stackelbergh equilibrium (SE)[11]. If a single user deviate from the equilibrium but utility does not change then the equilibrium is called nash equilibrium (NE). If single point satisfy both the equilibrium conditions then power allocation among the users minimized. We also consider bit error rate constraints for three modulation schemes. The modulation schemes are as follows: quadrature amplitude modulation (QAM), amplitude shift keying (ASK) and Phase shift keying (PSK) scheme with an adaptable modulation order M. Thus quality of service is assured. This is our main goal. In previous works either QAM or PSK modulation scheme is studied in [9] and [12].

So, in this paper our contributions is as follows:

- Minimize Problem search space, Satisfy QoS (BER) and solve linear and nonlinear relation by Game theory for three modulation scheme QAM, ASK, PSK and compare their performances.
- Gives user based distributed solutions thus computational complexity reduces.

III. GAME AND SYSTEM MODEL

A. STACKELBERG GAME

There are two types of players in Stackelberg strategic game. They are as follows: leaders and followers. The leader moves first and then the follower moves next. Therefore, knowing the leaders move, the follower chooses a strategy to optimize its own objective function (e.g. utility function). This is lower level.

By predicting the optimal response of the follower the leader, thus, has to optimize its own objective function (e.g. utility function). This is upper level of solution.

As in our power allocation problem CRs choose their requested SINR based on the unit-prices previously announced by BSs, we conclude that the strategies of CRs depend on strategies of BSs that move first. So, the Stackelberg game suits for modeling our problem best[7].

B. SYSTEM MODEL

We consider non-orthogonal multiple access scheme e.g. CDMA applied to a underlay cognitive radio network with a total of M secondary CRs and L channels in a typical cell. There are some assumptions about system model, (1) each channel can be used simultaneously by multiple secondary CRs via some form CDMA access scheme; (2) a single secondary user can use multiple channels at the same
time to meet their rate requirements; (3) every active SU radio has an upper limit on power and rate (bits/channel use) at which it can receive; (4) all SUs employ quadrature amplitude modulation (QAM), amplitude shift keying (ASK) and Phase shift keying (PSK) scheme with an adaptable modulation order M; (5) simple path loss model for channel has been assumed; (6) each user has a minimum rate and BER constraint that needs to be maintained and (7) an interference temperature threshold to protect possible primary user transmission on any channel.

The strategy of each BS is to determine the unit-price for each of its CRs and strategy of each CR is to determine the expected SINR level. Each BS tends to earn more utility from the CRs in its cell while paying less transmit power to them. Each CR is interested in gaining higher SINR level (or equivalently data rate) which paying less to the BS of its cell[7]. As a result, the utility functions of BS and CRs can be defined as follows:

$$U_{BS} = \sum_{k=1}^{L} \sum_{i=1}^{N_k} \Pi_i \gamma_i(k) - \sum_{k=1}^{L} \sum_{i=1}^{N_k} p_i(k)$$

Where $$\Pi_i$$ is the unit-price determined by BS for charging CRs, and $$\gamma_i(k)$$ is level of provided received SINR for which is a function of p (the vector of powers). The utility function of each CR can be written as:

$$U_{CRi} = \sum_{k=1}^{L} b_i(k) - \sum_{k=1}^{L} \gamma_i(k)$$

where

$$b_i(k) = \log(1 + \gamma_i(k))$$ and

$$\gamma_i(k) = \frac{p_i(k)h_{ij}(k)}{\sum_{j=1}^{N_k} \gamma_j(k)h_{ij}(k)p_{j}(k) + \sigma^2(k)}, \forall i, k$$

<table>
<thead>
<tr>
<th>NOTATIONS</th>
<th>EXPRESSION</th>
</tr>
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<tbody>
<tr>
<td>$$\Pi_i$$</td>
<td>Pricing factor of $$i$$-th user that is broadcasted by BS</td>
</tr>
<tr>
<td>$$\gamma_i(k)$$</td>
<td>SINR per bit for $$i$$-th user in $$k$$-th channel</td>
</tr>
<tr>
<td>$$N_k$$</td>
<td>Number of users for $$k$$-th channel</td>
</tr>
<tr>
<td>$$\sigma^2(k)$$</td>
<td>Orthogonality factor between users $$j$$ and $$i$$</td>
</tr>
<tr>
<td>$$h_{ij}(k)$$</td>
<td>Power gain from $$i$$-th transmitter to $$j$$-th receiver in $$k$$-th channel</td>
</tr>
<tr>
<td>$$h_{ij,m}(k)$$</td>
<td>Power gain from $$i$$-th transmitter at location $$m$$ in $$k$$-th channel</td>
</tr>
</tbody>
</table>

IV. RESOURCE ALLOCATION FRAMEWORK

Power and rate constraints of secondary CRs described in [9] has a centralized optimal solution. But it cumbersome and computationally ineffective. We have converted this to a game which is user based distributed approach. Here the set of power and rate of BS(leader) and CRs(followers) are nonempty, compact and approximately convex. In (1) and (2) both the utility functions are linear which can be considered as concave. Hence we have existence of a Stackelberg NE. It is simple to show that distributed algorithms, which the system can converge to Stackelberg equilibrium. In [10]-[11] the authors showed convergence of CRs to Stackelberg NE using the concept of a potential non-cooperative game.

It is easy to verify that the power allocation played by CRs and BS is a potential game with potential function in (1) and (2) which they try to maximize,

$$f(x) = \sum_{k=1}^{L} b_i(k) - \Pi_i \sum_{k=1}^{L} \gamma_i(k)$$

$$f(x) = \sum_{k=1}^{L} \sum_{i=1}^{N_k} \Pi_i \gamma_i(k) - \sum_{k=1}^{L} \sum_{i=1}^{N_k} p_i(k)$$

The strategy spaces of CRs and the power game of the BS are compact and convex sets and the potential function $$f$$ is continuous. Hence there exists at least one NE for the game for each price $$\Pi_i$$. Also the best response iteration converges to a Nash point[7] and [13].

V. PRACTICAL SOLUTION

A. GAME FORMULATION

The formulation of the proposed Stackelberg game can be decomposed into two levels: lower level of the CRs and upper level for the BS.
1) Lower level: Given the BS’s transmit power, the CRs’ non-cooperative sub-game can be formulated as strategic non-cooperative game.

In CR networks, the CR transmitters interact with each other. Hence, game theory suits best to analyze the behavior of the system. In game theory, the QoS that cognitive user received is referred to as utility function and cognitive users are the players of the game that finds to maximize their utility. The formal non-cooperative power control game of CR system can be defined as follows.

\[
G = \left[ \Omega_i \{p_i, \{b_i, \{U_i(\cdot)\}\} \right]
\]

where \( \Omega = \{1, 2, 3, \ldots, N\} \) is the players (CRs) index set, \( p_i = [0, p_i^{\text{max}}] \) represents the transmission power strategy set of user \( i \), and \( p_i^{\text{max}} \) represents the maximum transmission power of user \( i \). \( b_i = [1, b_i^{\text{max}}] \) represents the transmission bit rate strategy set of user \( i \), and \( b_i^{\text{max}} \) represents the maximum transmission bit rate of user \( i \). Conventionally, a single parameter formulates the search space. But in this game the search space constitutes two parameters and the whole space is the product of power and rate. For ease in presentation, we define the action for user \( i \) as

\[
y_i = \left[p_i^T b_i^T \right]^T,
\]

where \( (p_i = [p_i(1), p_i(2), \ldots, p_i(L)]^T \) and.

\[
b_i = [b_i(1), b_i(2), \ldots, b_i(L)]^T
\]

We consider utility function of user \( i \) as in (2)

\[
U_i(y_i, y_{-i}) = \sum_{k=1}^{L} b_i(k) - \Pi_{k=1}^{L} \gamma_i(k)
\]

where \( y_{-i} \), is the union set of all other users actions and \( y_{-i} = \left[y_1^T \ldots y_{i-1}^T, y_{i+1}^T \ldots y_M^T \right]^T \). The non-cooperative game formulation to determine transmit power and rate can be formally stated as

Determine \( y_i \)

To Maximize \( U_i (y_i, y_{-i}) \)

Subject to

CG1: \( 0 \leq p_i(k) \leq p_i^{\text{max}}(k), \quad \forall i, k \)

CG2: \( 1 \leq b_i(k) \leq b_i^{\text{max}}(k), \quad \forall i, k \)

CG3: \( \frac{1}{k} \sum b_i(k) \geq R_i, \quad \forall i \)

CG4: \( -\gamma_i(k) \leq -C_q \max \left(2b_i(k) - 1\right), \quad \forall i, k. \)

From the resource allocation framework [6] the system constraints formulate non-cooperative game. We take a conservative approach to satisfy those constraints in the game formulation.

First assumption the total interference caused by all CRs in a channel is divided equally across all CRs in that channel. This approach results in changing maximum limit on transmit power for each CR. In (13), this is captured in constraint CG1. In the game \( G \), a maximum limit on power

\[
p_i^{\text{max}}(k) = \min\left(p_i^{\text{max}}(k), \text{Upper_bound}_p \right)
\]

and the upper bound obtained from bound corresponds to \( I_{\alpha,k}(h_{\alpha,\text{TM}}(k)) \) for location \( m \). As an example using Table V

\[
p_i^{\text{max}}(k) = \min(6, I_{\alpha,k}(h_{\alpha,\text{TM}}(k)))
\]

Second assumption total supported rate in a channel is also divided across all CRs in that channel. This approach results in changing maximum limit on possible rate for each CR. In (8) this is captured in constraint CG2 in the game \( G \), the upper bound on maximum limit on rate

\[
b_i^{\text{max}}(k) = \min\left(b_i^{\text{max}}(k), \text{Upper_bound}_b \right)
\]

and the upper bound obtained from bound corresponds to \( R_{ch}(k) / (N_i(k)) \). As an example using Table V

\[
b_i^{\text{max}}(k) = \min(5, R_{ch}(k) / (N_i(k)))
\]

Here \( C_q \) calculated from the following BER formula for the modulation scheme using Table V for QAM

\[
P_{e,d}(k) = \frac{4}{b_i(k)} Q \left[ \frac{3b_i(k)\gamma_i(k)}{2b_i(k) - 1} \right], \quad \forall i, k, \text{odd} b_i(k); \quad (13)
\]

\[
P_{e,d}(k) = \frac{4}{b_i(k)} Q \left[ \frac{3b_i(k)\gamma_i(k)}{2b_i(k) - 1} \right], \quad \forall i, k, \text{even} b_i(k);
\]

for ASK
Here, $Q(x)$ is defined as $\int_{x}^{-\infty} e^{-\frac{z^2}{2}} dz$

In summary, based on local information, in order to minimize total power consumption, maximize rate, and maintain QoS (BER), we formulate a game to determine suboptimal distribution of power and rate that a secondary user has to employ across the channels.

2) Upper level: For the BS, if it can be CRs’ expected reactions to its action, we are to maximize the utility functions (1) of BS.

B. ANALYSIS OF THE GAMES

Algorithm 1 Algorithm to reach Stackelberg NE for the game $G$

**Input:** $t^\text{max} \in M, L, p_i^\text{max}(k), b_i^\text{max}(k), p_{e,j}^\text{th}$ and $h_{ch}^\text{th}(k)$;

**Output:** $p_i^t$;

Stopping counter, $t = 1$;

while ($t \leq t^\text{max}$ or $\|p_i^t - p_i^{t-1}\| / \|p_i^{t-1}\| \leq \varepsilon$), $\forall i$) do

% Execute optimization problem

for $i = 1, 2, ..., M$ do

for $k = 1, 2, ..., L$ do

Measure the interference and noise power

\[ (i.e., \sum_{j=1}^{M} p_j^{t-1}(k) h_{j,i}(k) p_j^2 + \sigma^2(k)) \]

across the intended channels; end for

Solve optimization problem (8) and obtain $p_i^t$ and $b_i^t$; end for

for $i = 1, 2, ..., M$ do

Transmit $p_i^t$;

end for

$t = t + 1$;

end while

Comparing distributed approach with centralized scheme we find that the user-based distributed approach is more attractive than centralized scheme in terms of information exchange requirement. As the centralized scheme requires information about all users and channels in the network it incurs a high communication overhead and poor scalability in CRN with large number of CRs. The required amount of information exchange in centralized scheme is $O(M^2)$. As a result, whereas, in the developed user-based distributed approach, each CR requires only local information- (i) possible number of users at next time instant $N_i(k)$, $\forall k$ and measurement of interference and noise power.

<table>
<thead>
<tr>
<th>TABLE II</th>
<th>SPECTRUM USAGE PATTERN ACROSS CHANNELS</th>
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<tbody>
<tr>
<td>Channel,</td>
<td>K</td>
</tr>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>User, 1</td>
<td>1</td>
</tr>
<tr>
<td>User, 2</td>
<td>1</td>
</tr>
<tr>
<td>User, 3</td>
<td>0</td>
</tr>
<tr>
<td>User, 4</td>
<td>0</td>
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<tr>
<td>User, 5</td>
<td>0</td>
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<td>User, 6</td>
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<td>User, 7</td>
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<td>User, 8</td>
<td>1</td>
</tr>
<tr>
<td>User, 9</td>
<td>1</td>
</tr>
<tr>
<td>User, 10</td>
<td>1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>TABLE III</th>
<th>CHANNEL QUALITY PARAMETERS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Channel, K</td>
<td>1</td>
</tr>
<tr>
<td>$\sigma^2(k), (\times 10^{-3})$</td>
<td>6</td>
</tr>
</tbody>
</table>

VI. NUMERICAL RESULTS

We study the performance of lower level of Stackelberg game which is a non-cooperative game of cognitive radio network. In the simulations, we assume that there are $L = 11$ available channels and a total of $M = 10$ secondary users. Table III provides information on the channel quality for all $L$ channels. Table IV lists the minimum rate requirement for each SU [4]. Finally table V contains all other system parameters that are relevant to our resource allocation framework.

<table>
<thead>
<tr>
<th>TABLE IV</th>
<th>MINIMUM RATE REQUIREMENT OF USERS</th>
</tr>
</thead>
<tbody>
<tr>
<td>User, i</td>
<td>1</td>
</tr>
<tr>
<td>$R_i$</td>
<td>9</td>
</tr>
</tbody>
</table>
Based on all this information, our objective is to find the optimal transmit power and rate that each of the CRs should employ to guarantee their QoS (BER) through our user-based distributed approach.

Fig 1. Allocation of total transmit power and total rate of CRs for ASK.

Fig 2. Comparison of total transmit power and total rate for Game Theory of CRs for QAM, ASK, and PSK respectively.

Fig 3. Comparison of total transmit power and total rate for Centralized Scheme of CRs for QAM, ASK, and PSK respectively.

Fig 4. Allocation of transmit power and rate with channel noise, variance and SINR for ASK.

Fig. 1(a) shows the allocation of total transmit power across users from both centralized from paper [9] and distributed (8) schemes, respectively. Fig. 1(b) shows the allocation of total rate across users from both centralized from paper [9] and distributed (8) schemes respectively. We see from Figs. 1(a) and 1(b) that both total allocated power and rate across users in distributed case are comparable to centralized scheme. Because in the game (8) maximum limit on power is set as the minimum of maximum usual limit on power and the upper bound. The upper bound can be obtained from dividing interference temperature threshold by the product of channel power gain at some location and possible number of users at next time instant as explained in (10). For the above approximations distributed solution gives less power than the counterpart. Additionally, our proposed distributed resource allocation scheme is successful in meeting minimum rate requirements for all CRs. The reason is obvious from the proposed user-based optimization problem formulation (8) after checking the
feasibility of the optimization problem solution. The feasibility is determined by user minimum rate requirement (constraint (CG3)). For each user, if the optimization problem is feasible, the distributed scheme is guaranteed to be successful in meeting the rate requirements for all CRs. Fig 1(b) clearly indicates that the proposed game statistics ensure the minimum rate requirement for all users.

Fig 2 presents the of total transmit power across users of distributed scheme for QAM, ASK and PSK modulation. From the constant $C_{qarg}$ calculated in [3] for QAM, ASK and PSK, we can predict that the QAM has the lowest power allocation and the PSK has the highest power allocation. Consequently the rate of three modulation schemes have the similar properties. Similar representation for centralized scheme is shown in Fig-3.

Fig.4 presents the transmit power and rate allocation across channels for user 5 from the proposed distributed scheme(8) along with centralized scheme in [9]. The channel noise variance and resulting SINR are also shown for reference. User 4 operates on channels 2, 3, 4, 5, 6, 7, 8, 9, 10 and 11. A game theory does not always give optimal result. We can observe the effect in the channel 6. Here game theory requires more transmit power.

Fig. 5(a) and 5(b) show the resulting total interference power and allocation of total rate, respectively, across channels from distributed scheme along with upper limits. We see from Figs 5(a) and 5(b) that both resulting total interference power and total rate across channels do not violate the corresponding upper limits. That is, the conservative approach based on constraints CG3 and CG4 in the proposed distributed case is successful in satisfying the system constraints in [9].

Upper limits in channels are used for fairness. The main purpose is to satisfy the requirement of individual users which is shown in fig: 1 to be satisfied.

VII. CONCLUSION
Nowadays Game theory based distributed approach is used for its easier implementability. In this paper, we consider the problem of power allocation to maximize utilities of users (BS and CRs) in a cognitive radio network that is suitable for Stackelberg Game. Using game theory, we propose a distributed power control scheme to protect primary users from interference while providing maximum possible utilization of base stations and cognitive radios. In addition, the received SINR level of CRs determined by the level requested by each individual CR. The proposed utility functions have led to a distributed architecture which is more realistic model. Numerical results are presented to prove the effectiveness of the proposed distributed algorithm.

REFERENCES


Fig. 5. Total interference power and total rate across channels

Fig. 5. Total interference power and total rate across channels