Artificial Bee Colony based Optimal PMU Placement in Power System State Estimation

Mohammad Shoaib Shahrir1*, Farhan Ammar Ahmad1, Ibrahim Omar Habibullah1, Mohammed Afzal Asif2, Shagorika Mukherjee2
1Department of Electrical Engineering, King Fahd University of Petroleum & Minerals (KFUPM), Dhahran, Saudi Arabia
2Department of Electrical and Electronic Engineering, Independent University, Bangladesh
*shoebeec05@gmail.com

Abstract—Advanced power system nowadays demands to ensure accurate estimate of the system states (bus voltages and angles) in order to ensure protection, monitoring, controlling and smooth running of the system. The conventional state estimator estimates the power system states based on the measurements obtained from the Supervisory Control and Data Acquisition (SCADA) system. Besides, the use of synchronized phasor measurement unit (PMU) has become a popular choice of the time in this field because of its ability of providing the real time phasors of voltage and currents. For a better phase estimation accuracy of the system, proper site selection of placing PMU is a must. Keeping an eye on system observability and system meter economy, it is important to identify the optimal location and number of PMUs to be used in state estimators. This paper presents an optimal solution of PMU’s placement by using one of the most widely used intelligent technique, Artificial Bee Colony (ABC). State estimation is then carried out by placing PMUs on the resultant optimum locations. Both the voltage and current phasors have been taken from the PMUs installed in the buses. The impact of bad data in the measurement series is also investigated. IEEE 14-bus and 30-bus systems has been taken as test system and Weighted Least Square (WLS) Algorithm has been considered as estimator algorithm to carry out the estimation process.

Keywords—State Estimation, Phasor Measurement Unit, Optimal PMU placement, Artificial Bee Colony.

I. INTRODUCTION

State estimation is such kind of an imperative process which does the very important duty of ensuring power system security by monitoring it precisely. A state estimator estimates the values of state variables (voltage magnitudes & phase angles) at the buses after it is provided with the measurements like magnitudes voltages and power injections in the buses, power flows and current flows in the branches [1-2]. Conventional Supervisory Control and Data Acquisition (SCADA) of remote terminal units (RTU) installed at the substations provides the estimator with these measurement values [3]. After introducing Phasor Measurement Unit (PMU) around the year 1990, it has become the most reliable tool in wide area monitoring system (WAMS) which provides real time bus voltage and branch current phasor measurements. The very high refreshing rate of PMU has made it highly acceptable in power system control compared with conventional SCADA systems [4]. Besides of state estimation, PMU has got some more applications like transient stability analysis of power system, fault detection, wide area protection, transmission line thermal monitoring etc. [5-7].

For proper running of any estimator, it must provide sufficient amount of measurements that make the system observable. The minimum number of measurements provided must be (2N-1) if N indicates the number of buses. Lots of work has already been done on this field and researchers has used both the heuristic and mathematical algorithms as solution methodologies [8-9]. Integer programming and exhaustive search technique was used to search for best placement location of PMUs which has been explained in detail in the taxonomy work on PMU placement done by Nikolas et al. [10]. Among the heuristic techniques, Tabu search, Simulated Annealing, Genetic Algorithm, Particle Swarm Optimization, Differential Algorithm, Bee colony, Spanning Tree Search, Immune Algorithm, Recursive Security N algorithm and several hybrids of such intelligent techniques were used in Optimum PMU placement (OPP). The recent most survey on OPP by Nazari-Heris et al. [11] presented all such techniques along with the solution sets for each as well as the objective functions considered in those problem formulation. All of those works mainly focused on minimizing the number of PMUs to be installed in the system which will make the system observable as well as will fulfill several other specified objectives.

This paper presents a technique to find out the optimal PMU locations in a power system. As it is known, that a bus if installed with a PMU, it can provide the voltage phasors of that bus as well as the current phasors of all or some of the adjacent branches connected to that bus depending upon the communication facility availability in that bus location. A MATLAB based optimization tool along with Artificial Bee Colony (ABC) is used to solve the optimization problem. Different solution sets are found for OPP which has been further investigated by doing state estimation. Performance of the estimator is checked by placing PMUs in the optimal solution sets keeping the measurement redundancy level same for all which will enable us to compare between the solution sets. Test cases with and without presence of bad data are considered both for estimation. Widely used WLS algorithm is used to carry out the state estimation for IEEE 14-bus system.

In this paper, section II will present the details of optimization problem of PMU placement. Section III will present the two optimization tools used in getting optimal solutions. WLS algorithm will be briefly presented in section...
IV. Part V will present all the simulation results with necessary analysis and followed by the conclusion in section VI.

II. PROBLEM FORMULATION FOR OPTIMAL PMU LOCATION

A PMU is different from the conventional meters in the sense that it has got the ability to provide not only the voltage phasors of the connected bus but also the current phasors of the branches which are attached with the bus [12-13]. In this problem formulation, it has been considered that the buses are all equipped with proper communication facilities which will allow an installed PMU on a particular bus to make all its neighboring buses observable. So, the problem formulation of OPP has got the objective to get the minimal set of PMUs so that each of the buses are at least reached once by the installed PMUs.

The objective function of the optimization problem for an n-bus system is [12]:

$$\min \sum_{i=1}^{n} x_i c_i \quad (1)$$

Where $x_i$ is the decision variable which denotes that whether a bus is installed with a PMU or not. For a 14 bus system, we will have 14 values of $x$ starting from $x_1$ upto $x_{14}$. So, $x_i$ can be represented as:

$$x_i = \begin{cases} 
1, & \text{if PMU is installed} \\
0, & \text{Otherwise} 
\end{cases} \quad (2)$$

$C_i$ is the cost needed to install a PMU in a particular bus. It is considered that the installation costs will be same for each PMUs wherever it is installed.

The connectivity matrix can be seen from the figure and can be formulated from equation (2) as follows:

$$A_{n \times n} = \begin{bmatrix}
1, & \text{if } i = j \\
1, & \text{if } i \text{ and } j \text{ are connected} \\
0, & \text{otherwise}
\end{bmatrix} \quad (4)$$

$$X_{n \times 1} = [x_1, x_2, x_3, ..., x_n]^T$$; a vector with decision variables

And $b = [1 \ 1 \ 1 \ ... \ 1]^T_{n \times 1} \quad (5)$

For IEEE 14 bus system, the formulation is presented in figure 4.

Min $(x_1 + x_2 + x_3 + ... + x_n)$

Subject to $[A]_{14 \times 14} [x]_{14 \times 1} = [b]_{14 \times 1}$

From the constraint equation (3), all of the 14 equations can be formulated which will help the system to make the system observable.

Suppose for bus 1 observability equation will be:

$$x_1 + x_2 + x_3 \geq 1$$; which means that at least one PMU must have to be placed in any of these three buses (1,2,5) so that other two buses could reach the installed PMU with proper communication channel and make the zone around bus 1 observable. Like the same way all other equations are formed for each of the buses which will make the respective bus "reachable" from the neighboring buses. Thus the entire system will become observable by following all the 14 constraint equations.
III. OPTIMIZATION TOOLS FOR OPP

a) CVX:
To solve the convex optimization problems, CVX is widely used which works in Matlab environment. It transforms Matlab into a modeling structured language, allow to incorporate objective functions and constraints using standard Matlab expression syntax which allows the Matlab users to deal with the tool very easily[14].

b) Artificial Bee Colony (ABC)
Artificial Bee Colony (ABC) is a meta-heuristic global optimization algorithm based upon the smart hunting and practical behavior of drone flock. It is a promising method for engineering applications and since its invention in 2005 by D. Karabog, it is being used for numerical optimization, combinatorial optimization problems as well as unconstrained and constrained optimization problems [15]. User defined size of population and cycle number are used as control parameters. Abandonment criteria is also predetermined by the user. The ABC algorithm progresses in mainly four steps:

Initialization
A fixed set of solutions or food source positions are initialized by the scout bees which is denoted as $x_{mi}$, where the value of m is bounded within 1 to SN (population size). In this step, the controlling limits are also set. Arbitrary food sources produced initially within the predefined range can be acquired from the following equation [16]:

$$x_{mi} = l_{i} + rand(0,1)*(u_{i} - l_{i})$$  \hspace{1cm} (6)

$l_{i}$ and $u_{i}$ are the lower and upper bound of $x_{mi}$. After initialization, the set of solutions are forced to undergo through an iterative search process of the following three phases [17].

Employed Bees Phase
For each employed bee:
- Determine new food source position, $v_{m}$ using the expression:
  $$v_{mi} = x_{mi} + \phi_{mi} \left( x_{mi} - x_{ki} \right)$$  \hspace{1cm} (7)
where $x_{ki}$ is a food source and i is a parameter index.
  $\phi_{mi}$ is a random number with the range [-a, a].
- Calculate the value $fit_m$ using:
  $$fit_m = \begin{cases} 
  \frac{1}{1 + f_m} & f_m \geq 0 \\
  \frac{1}{1 + abs(f_m)} & f_m < 0 
\end{cases}$$  \hspace{1cm} (8)

The m-th fitness value is the $fit_m$ and $f_m$ is the objective function value of the m-th solution.
- $v_m$’s with better fitness values than the corresponding $x_m$’s are used in updating the $x_m$’s. It is done by using a greedy selection.

Onlooker Bees Phase
For each onlooker bee:
- Food source (solution) with better probability value $p_m$, calculated based on the fitness values provided by the employed bees. The probability of an individual being selected by an onlooker bee is given by [16]:
  $$p_m = \frac{fit_m}{\sum_{m=1}^{SN} fit_m}$$  \hspace{1cm} (9)
- Determine new neighboring food source positions using equation (7)
- Calculate the fitness value, $fit_m$
- Similarly, as in employed bee phase, the best individual is being updated after each iteration based on fitness values.

Scout Bees Phase
The employed bees which cannot improve its solution by using preset number of trials (known as limits or terminating condition) become scout and as such their solutions get discarded. Thus, the scouts get busy looking for new solutions in a random manner and is defined by equation (6) [18].

IV. STATE ESTIMATION USING WLS ALGORITHM
Weighted Least square represents an estimation problem that selects the criterion of a solution to the over-defined matrix equations when less states and more measurements are available in the power system [1-2]. Weighted least square have the objective function that minimizes the square of the error. Traditional state estimator utilizes SCADA measurements whose relationship with system states:
\[ z_i = h_i(x) + e_i \]  
(10)

\[ z_i \] is SCADA measurement vector and its size is \((m \times 1)\)

\[ h_i(x) \] correspond to a nonlinear function vector which relates measurements to states

\[ e_i \] is the error vector between estimated and measured values of size \((m \times 1)\)

Power System State Estimation is a system of overdetermined nonlinear equations and must be solved as unconstrained WLS problem. For weighted least square we need to minimize sum of square of residuals:

\[
\min f(x) = \sum_{i=1}^{N} W_i^2 \| h_i(x) - z_i \|^2
\]

(11)

\[
\min f(x) = \frac{1}{2} \left[ z - h(x) \right]^T R^{-1} \left[ z - h(x) \right]
\]

(12)

By taking partial derivative of \( h(x) \) with respect to state vector \( x \), Jacobian matrix \([H]\) will be obtained.

\[
H = \begin{bmatrix}
\frac{\partial P_{flow_{[u]}}}{\partial \theta} & \frac{\partial P_{flow_{[u]}}}{\partial v_m} \\
\frac{\partial P_{flow_{[u]}}}{\partial v_m} & \frac{\partial P_{flow_{[u]}}}{\partial v_m} \\
\frac{\partial Q_{flow_{[u]}}}{\partial \theta} & \frac{\partial Q_{flow_{[u]}}}{\partial v_m} \\
\frac{\partial Q_{flow_{[u]}}}{\partial v_m} & \frac{\partial Q_{flow_{[u]}}}{\partial v_m} \\
\frac{\partial I_{F,\text{Real}_{[u]}}}{\partial \theta} & \frac{\partial I_{F,\text{Real}_{[u]}}}{\partial v_m} \\
\frac{\partial I_{F,\text{Real}_{[u]}}}{\partial v_m} & \frac{\partial I_{F,\text{Real}_{[u]}}}{\partial v_m} \\
\frac{\partial I_{F,\text{Imag}_{[u]}}}{\partial \theta} & \frac{\partial I_{F,\text{Imag}_{[u]}}}{\partial v_m} \\
\frac{\partial I_{F,\text{Imag}_{[u]}}}{\partial v_m} & \frac{\partial I_{F,\text{Imag}_{[u]}}}{\partial v_m}
\end{bmatrix}
\]

According to Newton method to minimize a function \( f(x) \), where \( x^{k+1} = x^k + \Delta x \) is

\[
\Delta x = \left[ f'(x) \right]^{-1} f(x) = \left[ \frac{\partial V_{f(x)}}{\partial x} \right]^{-1} \left[ -\nabla f(x) \right]
\]

where

\[
\frac{\partial V_{f(x)}}{\partial x} = 2[H]^T[R]^{-1}[H]
\]

And the complete formulation for updating states becomes

\[
\begin{bmatrix}
z_1 - h_1(x) \\
z_2 - h_2(x) \\
\vdots \\
z_m - h_m(x)
\end{bmatrix} = x_{k+1} = x_k + \left[ [H]^T[R]^{-1}[H] \right]^{-1} \left[ [H]^T[R]^{-1} \right] \left[ z_i - h_i(x) \right]
\]

\( \Delta x \) is the measurement mismatch, which is used as iteration step for next iteration, for \( \Delta x \) to exist non-singularity for gain matrix \([H]^T[R]^{-1}[H]\) is must. \( R \) (error covariance matrix of SCADA measurements), iterative procedure terminates when \( \Delta x \) goes below a certain low threshold value.

V. SIMULATION RESULTS

A) IEEE 14 bus system:

As discussed above, simulation is carried out on the IEEE 14 bus test system. In solving the optimization problem in placing PMUs in the proper location, MATLAB based optimization tool CVX is applied for the comparison with the proposed Artificial Bee Colony (ABC) algorithm. Three different optimal solution sets are found from ABC optimization with same number of PMUs (four) which can meet the objective function. The minimum number of PMUs required and the best bus locations for PMU placements are presented in table I below for both the optimization tools.

| Table I: Results for Optimal PMU locations |
|-----------------|-----------------|-----------------|
| Optimized with  | Number of PMUs  | PMU locations   |
| CVX             | 5               | 2, 6, 7, 9, 13  |
| Artificial Bee Colony (ABC) | 4 | Set1: 2, 6, 7, 9 |
|                 | 4               | Set2: 2, 7, 11, 13 |
|                 | 4               | Set3: 2, 7, 10, 13 |

It can be observed from table I that ABC is giving better results with keeping the number of PMUs only 4 even though they are having some different sets of solutions. CVX optimized the problem with 5 PMUs. In all three sets of solutions given by ABC, the system has become observable.

1) Without any Bad data:

The test case has been made of measurement readings taken from the conventional meters. PMUs are then placed in the obtained optimal locations as presented in table I and state estimation is carried out then to see their impact on the estimation performance. This will lead to find out the optimal solution set. It has been done for each of the solution sets with same redundancy level of measurement values so that a comparison can be made between the solution sets. To make
the test cases, total of 42 measurements are taken with a redundancy level of 1.55. To represent the estimator performance, an indicator has been used which is actually the sum of the differences between all the estimated values and the base case values. Lower value of indicator indicates the better performance of the estimator.

Initially, only the voltage magnitudes and phasors from PMUs are considered to run the estimation. So, for the case of 4 PMUs, we are considering 8 measurement values taken from PMU meters and rest are from conventional SCADA meters.

After that the estimation is carried out with considering the current phasors from PMUs. To keep the redundancy level same, 8 conventional power flow readings are replaced by same amount of current measurements of PMUs. The results obtained are presented in table II below.

<table>
<thead>
<tr>
<th>Optimized with</th>
<th>PMU locations</th>
<th>Measurement Redundancy</th>
<th>Estimator Indicator</th>
</tr>
</thead>
<tbody>
<tr>
<td>CVX</td>
<td>2, 6, 7, 9, 13</td>
<td>1.55</td>
<td>3.4668</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2.8523</td>
</tr>
<tr>
<td>Artificial Bee Colony (ABC)</td>
<td>Set1: 2, 6, 7, 9</td>
<td>1.55</td>
<td>3.4096</td>
</tr>
<tr>
<td></td>
<td>Set2: 2, 7, 11, 13</td>
<td>1.55</td>
<td>3.4283</td>
</tr>
<tr>
<td></td>
<td>Set3: 2, 7, 10, 13</td>
<td>1.55</td>
<td>3.3529</td>
</tr>
</tbody>
</table>

It is seen from the results that the estimator performance for CVX solution set is not better than ABC even if the PMU numbers are higher. Inclusion of current phasors improved the estimator performance for all the cases as the indicators are much lower when current phasors are included. Results show that the solution set3 for ABC with PMU locations in buses 2, 7, 10, 13 gives the best performance indicator with lowest value.

2) With bad data:

A single bad data is then considered in the measurements series. Power flow of branch 4 to 7 has been taken -0.3526 now which was 0.3526 previously. Because of the presence of this bad data, indicator becomes 8.2919 which was 4.8916 before. Table III clearly indicates that the third solution set of ABC algorithm with PMU locations 2, 7, 10, 13 gives the best solution with lowest indicator among all. The results are presented below in table III.

Inclusion of current phasors in the place of power flows improved performance as it happened without bad data too. The solution set of CVX, even providing an extra PMU to be installed, could not perform that well in estimation. Placing an extra PMU will not be an economical choice too.

Similarly, like 14 bus, state estimation is then carried out by WLS algorithm after placing PMUs in each of the optimal location sets.

<table>
<thead>
<tr>
<th>Optimize d with</th>
<th>PMU locations</th>
<th>Measurement Redundancy</th>
<th>Estimator Indicator</th>
<th>Voltage from PMU</th>
<th>Voltage and current from PMU</th>
</tr>
</thead>
<tbody>
<tr>
<td>CVX</td>
<td>2, 6, 7, 9, 13</td>
<td>1.55</td>
<td>7.3238</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Artificial Bee Colony (ABC)</td>
<td>Set1: 2, 6, 7, 9</td>
<td>1.55</td>
<td>5.2336</td>
<td>5.0382</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Set2: 2, 7, 11, 13</td>
<td>1.55</td>
<td>5.2862</td>
<td>5.0217</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Set3: 2, 7, 10, 13</td>
<td>1.55</td>
<td>5.2014</td>
<td>4.9914</td>
<td></td>
</tr>
</tbody>
</table>

B) IEEE 30 bus System:

The work is then extended to the larger power system with 30 buses. The superiority of ABC over CVX in optimizing problems is being proved already in previous section. Different optimal solution sets by Artificial Bee Colony algorithm for placing PMUs are presented below in table IV.

**Table IV:** Results for Optimal PMU locations by ABC

<table>
<thead>
<tr>
<th>Number of PMUs</th>
<th>PMU locations</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>Set1: 3, 5, 8, 10, 12, 18, 19, 24, 25, 29</td>
</tr>
<tr>
<td>9</td>
<td>Set2: 1, 7, 8, 10, 12, 19, 23, 26, 29</td>
</tr>
<tr>
<td>9</td>
<td>Set3: 1, 7, 8, 10, 12, 19, 24, 26, 30</td>
</tr>
<tr>
<td>9</td>
<td>Set4: 1, 5, 10, 11, 12, 19, 24, 26, 27</td>
</tr>
<tr>
<td>9</td>
<td>Set5: 3, 5, 8, 10, 12, 18, 23, 26, 29</td>
</tr>
<tr>
<td>9</td>
<td>Set6: 3, 5, 8, 10, 12, 19, 23, 25, 29</td>
</tr>
<tr>
<td>9</td>
<td>Set7: 3, 5, 8, 10, 12, 15, 20, 25, 29</td>
</tr>
<tr>
<td>9</td>
<td>Set8: 1, 7, 8, 10, 12, 15, 19, 25, 29</td>
</tr>
<tr>
<td>9</td>
<td>Set9: 1, 5, 10, 11, 12, 19, 23, 25, 27</td>
</tr>
</tbody>
</table>

1) Without any Bad data: Test case has been made with the same redundancy level (1.55) as done for 14 bus system. Results are presented below in table V.

**Table V:** SE indicator with readings from PMUs

<table>
<thead>
<tr>
<th>PMU locations</th>
<th>Voltage from PMU</th>
<th>Voltage and current from PMU</th>
</tr>
</thead>
<tbody>
<tr>
<td>Set1: 3, 5, 8, 10, 12, 18, 24, 25, 29</td>
<td>12.0423</td>
<td>7.7182</td>
</tr>
<tr>
<td>Set2: 1, 7, 8, 10, 12, 19, 23, 26, 29</td>
<td>12.4724</td>
<td>8.0055</td>
</tr>
<tr>
<td>Set3: 1, 7, 8, 10, 12, 19, 24, 26, 30</td>
<td>12.3997</td>
<td>8.6287</td>
</tr>
<tr>
<td>Set4: 1, 5, 10, 11, 12, 19, 24, 26, 27</td>
<td>11.9904</td>
<td>8.4069</td>
</tr>
<tr>
<td>Set5: 3, 5, 8, 10, 12, 18, 23, 26, 29</td>
<td>12.3182</td>
<td>8.7552</td>
</tr>
<tr>
<td>Set6: 3, 5, 8, 10, 12, 19, 23, 25, 29</td>
<td>12.0081</td>
<td>8.2738</td>
</tr>
<tr>
<td>Set7: 3, 5, 8, 10, 12, 15, 20, 25, 30</td>
<td>12.0463</td>
<td>8.2971</td>
</tr>
<tr>
<td>Set8: 1, 7, 8, 10, 12, 15, 19, 25, 29</td>
<td>12.1249</td>
<td>8.2745</td>
</tr>
<tr>
<td>Set9: 1, 5, 10, 11, 12, 19, 23, 25, 27</td>
<td>11.7615</td>
<td>8.0818</td>
</tr>
</tbody>
</table>

Only the voltage phasors are considered initially from PMUs. Current phasors are then considered by replacing same amount of power flows to keep the redundancy same. It is seen that inclusion of current phasors improves estimation indicator for all cases and it also changes the optimal solution set. PMU location set 9 was best among all when only voltage phasors were considered. Inclusion of current phasors from PMUs made solution set 1 the best one with lowest indicator 7.7182.
The optimal solution sets are similar as the cases of without applying bad measurement. Voltage measurements only from PMUs give optimal solution set 9 as the best solution with least indicator of 17.4219. Incorporation of PMU current measurements results solution set 01 as the best optimal locations as it gives the minimum indicator 7.7182.

VI. CONCLUSION

This paper focused on two main issues: to place PMUs optimally which will not only reduce the installation cost but also make the overall system observable and to check the performance of the state estimator if the PMUs are placed in the resultant optimal positions irrespective of the presence of bad data. It has been found that Artificial Bee Colony based optimal position provides the best estimation performance. For the case of running estimation, redundancy has been taken considerably above 1 which will allow the estimator to work even if some meters fail to provide service. Besides of PMU readings, some measurement readings has been taken from conventional meters with some sort of normal noises. The detail formulation of optimization problem has been presented as well as the formulation of the estimator. It has been observed that the performance of the estimation process was improved when using ABC by indicating the optimal placement of PMUs in the system. The results carried out using the IEEE 14-bus and 30 bus system.

REFERENCES


Table V: SE indicator with readings from PMUs after applying bad data

<table>
<thead>
<tr>
<th>PMU locations</th>
<th>Estimator Indicator</th>
</tr>
</thead>
<tbody>
<tr>
<td>Voltage from PMU</td>
<td>Voltage and current from PMU</td>
</tr>
<tr>
<td>Set1: 3, 5, 8, 10, 12, 18, 24, 25, 29</td>
<td>17.7770</td>
</tr>
<tr>
<td>Set2: 1, 7, 8, 10, 12, 19, 23, 26, 29</td>
<td>18.2584</td>
</tr>
<tr>
<td>Set3: 1, 7, 8, 10, 12, 19, 24, 26, 30</td>
<td>18.1936</td>
</tr>
<tr>
<td>Set4: 1, 5, 10, 11, 12, 19, 24, 26, 27</td>
<td>17.6991</td>
</tr>
<tr>
<td>Set5: 3, 5, 8, 10, 12, 18, 23, 26, 29</td>
<td>18.0779</td>
</tr>
<tr>
<td>Set6: 3, 5, 8, 10, 12, 19, 23, 25, 29</td>
<td>17.7401</td>
</tr>
<tr>
<td>Set7: 3, 5, 8, 10, 12, 15, 20, 25, 30</td>
<td>17.7910</td>
</tr>
<tr>
<td>Set8: 1, 7, 8, 10, 12, 15, 19, 25, 29</td>
<td>17.9509</td>
</tr>
<tr>
<td>Set9: 1, 5, 10, 11, 12, 19, 23, 25, 27</td>
<td>17.4219</td>
</tr>
</tbody>
</table>

2) With Bad data:

Bad data is then applied in one of the power flow values. Estimator performance deteriorates as expected. Then the PMU measurements are considered as above and the results are presented below in table V.