Basic Sequential Algorithmic Scheme Based Blind Common Phase Error Compensation in OFDM Systems

Md. Alamgir Hossain
Khulna University of Engineering & Technology
Khulna-9203, Bangladesh
Email: mah@mail.kuet.ac.bd

Abstract—In this paper, Basic Sequential Algorithmic Scheme (BSAS) based blind Common Phase Error (CPE) compensation method is proposed to abate the effect of Phase Noise (PHN) in OFDM system to retain the system bandwidth. By using the proposed algorithm, all the signal points are grouped according to the constellation size and average angle of each group is estimated. The estimated average angles are compared with the corresponding ideal angles and averaged again to estimate CPE. The performance of the proposed method is demonstrated by several computer simulations using MATLAB.

Keywords: BSAS, OFDM, Phase Noise, Common Phase Error, Inter-carrier Interference, Phase Locked Loop.

I. INTRODUCTION

OFDM system has been widely implemented in various wireless and wireline communication systems, such as IEEE 802.11a/g, IEEE 802.16, HIPERLAN etc. It is vigorous against inter-carrier interference (ICI) and inter-symbol interference (ISI) caused by multipath frequency selective channel. But, the most vital aspect, the orthogonality among the sub-carriers, of OFDM system is intimidated by the presence of PHN. To compensate for the effect of PHN, several pilot based PHN estimation algorithms are proposed, such as in [1]-[3]. In most of the existing pilot based methods, some pilots are sent with each OFDM block to estimate the multi-path channel and these pilot are also used to estimate the phase noise. The exploitation of pilot symbols reduces the system bandwidth. Besides, due to the time-varying nature of PHN, the pilot sequence requires to be transmitted periodically, results further reduction of system throughput. In order to improve the bandwidth efficiency, blind methods for PHN compensation have attracted much attention. In [4]-[13] the authors have proposed a blind algorithm that suffers from higher computational complexity. In most of the existing blind algorithms, there is a performance losses arising from decision errors, since it uses the decision-directed approach [7]. Recent advancement in wireless communication, such as WiMAX has been adopted with one out of three consecutive OFDM symbols are allocated for pilot while in LTE one full OFDM is dedicated for channel estimation once every seven time slots. Therefore, it is crucial to detect data symbols with the help of pilot symbols, this motivation directs this research to develop an algorithm to estimate the data symbol as well as phase noise without pilot in each OFDM symbol. In this paper, BSAS based [8] blind PHN compensation method is proposed to abate the effect of CPE in OFDM system caused by PHN. In the proposed algorithm, the signal points are grouped according to the constellation size by exploiting the BSAS algorithm.

II. SYSTEM MODEL

A block diagram of OFDM transceiver system with PHN is shown in Fig. (1). The time-domain signal \( s[n] \) is obtained by applying \( N \)-point Inverse Fast Fourier Transform (IFFT) on frequency-domain signals, \( S[k] \) given by,

\[
s[n] = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} S[k] e^{j2\pi nk/N}
\]

(1)

where, \( k, n = 0, 1, \cdots N-1 \). a cyclic prefix (CP) of length \( N_c \geq L \) samples are added at the beginning of each OFDM block to prevent the effect of ISI, where \( L \) is the length of the multi-path channel. The carrier frequency offset and channel conditions are estimated in the training phase and compensated for in the data detection stage, hence the effect of frequency offset is minimized and the channel state information is assumed available at the receiver. After removing the CP, the received complex baseband signal of m^{th} OFDM block in the presence of PHN can be written as

\[
y_m[n] = e^{j\phi[n]} \sum_{i=0}^{L-1} h_m[i]s_m[n]\mod N + \nu_m[n],
\]

(2)

where, \( h \) is \( L \) taps baseband channel and \( \nu \) is AWGN noise. To demodulate the received time-domain signal, \( N \)-point FFT is applied on it and hence the frequency-domain signal \( Y_m[n] \)

Fig. 1. OFDM Transceiver System with PHN
is given by

\[ Y_m[n] = \sum_{l=0}^{N-1} H_m[l]S_m[l]C[l-n]N + Z_m[n] \]
\[ = \Phi_m[0]S_m[n]H_m[n] \]
\[ + \sum_{l=0, l\neq n}^{N-1} H_m[l]S_m[l]C[l-n]N + Z_m[n] \] (3)

It will discussed in the following sub-section that the PHN angle is small and can be expressed as \( e^{j\phi[n]} \approx 1 + j\phi[n] \).

Therefore,

\[ \Phi_m[0] \approx \frac{1}{N} \sum_{n=0}^{N-1} (1 + j\phi[n]) = 1 + \frac{j}{N} \sum_{n=0}^{N-1} \phi[n] = 1 + j\bar{\phi} \] (4)

where,

\[ \bar{\phi} = \frac{1}{N} \sum_{n=0}^{N-1} \phi[n] \]

is the angle of rotation caused by CPE. \( \Phi_m[0] \) in equation (3) depict the effect of CPE that causes the rotation of every received symbol by an average angle \( \bar{\phi} \) given by equation (4). The second term of equation (3) is ICI. It will be discussed later that the variation of generated PHN process is low and the effect of ICI is small compared to CPE. Besides, the ICI term can be approximated as zero-mean complex Gaussian noise, and can be merged with AWGN noise. So, the equation (3) can be rewritten as

\[ Y_k = \Phi_0 S_k H_k + \xi_k \] (5)

where, \( \xi_k \) consists of ICI and AWGN noise. The main objective of this research is to mitigate the effect of CPE, i.e., to estimate and compensate for the average angle that causes CPE to demodulate the received data bit efficiently.

### III. Phase Noise Model

#### A. Source of PHN

The output voltage of an ideal LO can be described mathematically as a pure sine wave of constant frequency and amplitude. In practice, this type of oscillator is impossible. A real oscillator output voltage is expressed in [9] given by

\[ v(t) = [A + \alpha(t)] \cos(2\pi f_c t + \phi(t)) \] (6)

where, \( \alpha(t) \) and \( \phi(t) \) are amplitude and phase noise modulation respectively. If the oscillator is well-designed, amplitude noise is less significant than phase noise. Practically, it is very difficult to implement this type of oscillators. The rapidly increasing demand of bandwidth leads to higher order and more compact carrier frequency overlapping modulation techniques. It gets pretty challenging to recover the transmitted data in the presence of phase noise. Since, the oscillator amplitude noise can be fixed-up by using an automatic gain controller (AGC), the main issue in this paper is to deal with the time-varying random PHN process.

The nature of PHN rely on the type of frequency generator that may be a free running oscillator or frequency synthesizer.

Most of the wireless communication systems use phase lock loop (PLL) based frequency synthesizer [10] due to its high level of stability, easy control, wide range of frequency generation as well as higher accuracy. PHN is generated from the non-ideal characteristics of PLL results PHN that is random process caused by the phase fluctuation of the oscillators.

Better hardware design may reduce the effect of PHN but improved estimation scheme is an important issue to estimate the PHN. Most of the existing research works assume Wiener model (Random walk) for phase noise [11]. Such a model is only suggested when the carrier signal is generated from a stand alone local oscillator. This type of design is almost never adopted as a free running oscillator that slowly drift away from the required phase and frequency. Whereas, the gain of a PLL is almost constant within the loop bandwidth and there is a 30dB/dec roll off outside the loop bandwidth.

The block diagram of a PLL is shown in Fig. (2) that consists of a reference signal source, a phase detector (PD), a low pass filter (LPF), a voltage controlled oscillator (VCO) and a frequency divider. The function of PD is to detect the phases of two input signals, then compare the phases and gives an error signal depending on the phase difference. If the reference input and the signal from divider are given by

\[ s_r = A_r \sin(\omega_c t + \phi_r(t)) \] (7)
\[ s_d = A_d \cos(\omega_c t + \phi_d(t)) \] (8)

then, the PD output will be

\[ s_p = s_r \times s_d = \frac{A_rA_d}{2} \left[ \sin(\phi_r(t) - \phi_d(t)) + \sin(2\omega_c t + \phi_r(t) + \phi_d(t)) \right] \] (9)

The high frequency component is filtered out by the LPF and the input voltage of the VCO is given by

\[ s_{LP} = \frac{A_rA_d}{2} \sin(\phi_r(t) - \phi_d(t)) \] (10)

where, \( \phi_r \) and \( \phi_d \) are the phase of reference and detector signals respectively. Using PLL and VCO, random phase noise is generated as shown in Fig. (3) that illustrates the PHN is a zero mean random variable.

#### B. Effect of PHN on OFDM

To validate the consequence of PHN on OFDM systems, computer simulations were carried out using the following parameters shown in section V. It is clearly seen from the scatter plot shown in Fig. (4) that the received signals, blue
Fig. 3. The Random PHN.

Fig. 4. Effect of CPE on 64-QAM OFDM marked (dot), are rotated counter-clockwise from its original location, the place of signal without phase noise marked in red (star), by an average angle $\bar{\phi}$. The main objective of this research is estimate this average angle of rotation and to mitigate the effect of CPE.

IV. CPE ESTIMATION

The signal points are rotated due to CPE and if the angle of rotation exceeds threshold angle, then it is quite difficult to detect the transmitted data symbols. Different threshold angles depending on different M-QAM constellations are discussed in [9]. In that case, different algorithms or special error correcting codes are necessary to efficiently detect the symbol. In this section, a CPE compensation algorithm is proposed to subside the effect of CPE in OFDM. In the proposed algorithm, the signal points are grouped according to the constellation size by exploiting BSAS algorithm. Then, the angles of signals of same group are calculated and compared with the idle angle and averaged to obtain phase error for that particular point. This process is repeated for all available constellation points and averaged again to estimate the angle of rotation caused by CPE. The performance of the proposed method is demonstrated by the computer simulations. PLL frequency synthesizer is used as the source of PHN [12], and for the simplicity, only the receiver phase noise is considered. The performance of the proposed algorithm is demonstrated by the computer simulations. To do this, 64-QAM modulation technique is exploited to modulate the frequency domain signal. Firstly, the conventional pilot based method of CPE estimation will be analyzed, and then the simple but robust BSAS based CPE estimation algorithm will be proposed.

A. Conventional Pilot Based Estimation

Suppose $S_p$ presents the set of indices corresponding to pilots in an OFDM signal. To estimate the CPE angle $\bar{\phi}$, the least-square (LS) method [3] is applied by minimizing the cost function,

$$\text{min}_{C_0} \sum_{k \in S_p} | Y_k - C_0 S_k H_k |^2$$

The pilot based estimation of CPE angle can be obtained [13] as follows-

$$\hat{\bar{\phi}}_\text{pilot} = \mathbb{I} \left( \sum_{k \in S_p} Y_k (H_k S_k)^* \right) \left( \sum_{k \in X_p} | H_k S_k |^2 \right)^{-1}$$

(B) Proposed BSAS Based Estimation

For time-varying nature of PHN, periodical pilot symbols are needed in each OFDM symbol at the cost of system bandwidth. This motivation leads this research to develop a blind algorithm to estimate PHN. The BSAS algorithm performs a single pass on a given data set. In addition, each cluster is represented by the mean of the vectors that have been assigned to it. For each new vector $s$, presented to the algorithm, its distance from the already formed clusters is computed. If these distances are larger than a (user-defined) threshold of dissimilarity, $\Theta$, and if the maximum allowable number of clusters, $q$, have not been reached, a new cluster containing $s$ is created. Otherwise, $s$ is assigned to its closest cluster and the corresponding representative is updated. The algorithm terminates when all data vectors have been considered once. The BSAS algorithm can be expressed as follows:

Let $m = 1$

$C_m = s_1$

for $i = 2$ to $N$,

Find $C_k : d(s_i, C_k) = \min_{1 \leq j \leq m} d(s_i, C_j)$

If $(d(s_i, C_k) > \Theta) \text{AND} (m < q)$,

$m = m + 1$

$C_m = s_i$

else

$C_k = C_k \cup s_i$

end

end

To find out the angle of rotation caused by CPE as shown
In Fig. (4), the received frequency-domain signal points were divided into groups according to the constellation size. From this group of signals, average angle w.r.t reference axis is estimated. The average angles estimated for all groups are expressed by

$$\phi_{av}(p) = \frac{1}{N_s} \sum_{j=1}^{N_s} \text{arg}\{C_j\}$$  \hspace{1cm} (12)$$

where, \(p = 1, 2, \ldots M\) be the constellation size. The averaged angles were compared with the corresponding ideal angles that gives \(M\) individual CPE angles,

$$\phi_{\text{cpe}}(p) = \phi_{\text{ideal}}(p) - \phi_{av}(p)$$ \hspace{1cm} (13)

These CPEs were again averaged to figure out the desired angle of rotation and given by the following cost function,

$$\phi_{\text{proposed}} = \frac{1}{M} \sum_{j=1}^{M} \phi_{\text{cpe}}(j)$$ \hspace{1cm} (14)

V. SIMULATION RESULTS

To investigate the performance of the proposed method, several computer simulations were performed by using the following system parameters; The total number of sub-carriers, \(N_s = 1024\), the data sub-carriers, \(N_D = 824\) and unused sub-carriers were 200. The data symbols were modulated by using 64-QAM. The known channel type was 10 tap Rayleigh fading multipath channel. the sampling frequency and 3dB bandwidth were 20MHz and 10kHz respectively. The rms value of PHN was \(3^0\). The simulation results are depicted in Fig. (5) and (6). In Fig. (5) the transmit and received symbols after estimation are compared and it is clearly seen that most of the symbols are decoded efficiently, whereas in Fig. (6) bit error rate performance of proposed method is compared with conventional pilot based method.

![Fig. 5. Comparison of Transmit and Received Symbols.](image)

VI. CONCLUSION

Though blind algorithm suffers from calculation complexity, to save the valuable bandwidth with the cost of lower calculation load an improved and bandwidth efficient new algorithm has been offered for CPE compensation in OFDM systems. Simulations show that the proposed Basic Sequential Algorithmic Scheme smash the conventional pilot based CPE compensation method.

REFERENCES


![Fig. 6. BER Performance Comparison](image)